

An absolute polarimeter for high energy protons

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A study of the spin asymmetries for polarized elastic proton-proton collisions in the electromagnetic hadronic interference [Coulomb nuclear interference (CNI)] region of momentum transfer provides a method of self-calibration of proton polarization. The method can be extended to non-identical spin half scattering so that, in principle, the polarization of a proton may be obtained through an analysis of its elastic collision with a different polarized particle, ^3He , for instance. Sufficiently large CNI spin asymmetries provide enough information to facilitate the evaluation of nearly all the helicity amplitudes at small t as well as the polarization of both initial spin half fermions. Thus it can serve equally well as a polarimeter for ^3He .

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I. INTRODUCTION

The Relativistic Heavy Ion Collider (RHIC) in its pp mode is a unique machine. It opens a new frontier in the use of spin for the study of hadronic physics. Using its polarized beams a whole new range of tests of the standard model will become feasible, and much new information about the detailed partonic structure of the nucleon will emerge [1]. It will also be possible to answer intriguing questions concerning the relationship between pp and $\bar{p}p$ total cross sections and real parts of forward amplitudes, questions which are relevant to attempts to understand certain aspects of nonperturbative QCD [2]. The entire rich program relies upon an accurate determination of the polarization of the proton beams, hopefully to an accuracy of $\pm 5\%$, a matter which is far from trivial.

Much effort has gone into attempts to devise a polarimeter with the required accuracy. The attractiveness of using a Coulomb nuclear interference (CNI) polarimeter to measure the polarization at very high energies stems from the belief, as folklore has it, that all hadronic helicity-flip amplitudes are negligible at high energies. In that case, and ignoring the real part of the non-flip amplitude, the analyzing power A_N has a maximum at $t = t_p \equiv -8\pi\sqrt{3}\alpha/\sigma_{\text{tot}}$ [3] and its value at the peak is

$$A_N^{\text{max}} = \frac{(-3t_p)^{1/2}}{2m} \left(\frac{\kappa}{2} \right) \quad (1)$$

where $\kappa = \mu - 1$ is the anomalous magnetic moment of the proton and m its mass. (At high energies A_N^{max} is between 4% and 5%.) Thus, in the above scenario, one has an almost perfectly calibrated polarimeter. The problem is that if the helicity flip amplitudes are non-negligible then $\kappa/2$ in Eq. (1) has to be replaced by

$$\frac{\kappa}{2} \rightarrow \frac{\kappa}{2} - I_5 + \frac{\kappa}{2} I_2 \quad (2)$$

where I_2 and I_5 essentially measure the size of the imaginary parts of ϕ_2 and ϕ_5 relative to the imaginary part of the dominant non-flip amplitude $\frac{1}{2}(\phi_1 + \phi_3)$. The precise definitions of I_2 and I_5 are given in terms of helicity amplitudes in Eqs. (8) to (11) below. The ϕ_i 's are defined in [4]. Thus the efficacy of the CNI polarimeter rests upon being able to demonstrate that I_5 and I_2 are negligible or knowing precise values for them. In a very comprehensive study of this issue [5] we concluded that while it was likely that I_2 can be neglected, it was not possible to exclude the possibility that $|I_5| \approx 15\%$ implying that the value of A_N^{max} cannot be calculated to the desired accuracy of $\pm 5\%$. (A study of Coulomb interference with polarized protons, from a rather different point of view, has been made by Jakob and Kroll [6].)

However, in the course of this study we discovered the extraordinary fact that pp elastic scattering is self-calibrating, in the sense that a measurement of a sufficient number of spin-dependent observables at very small t using a polarized beam and target, but without *a priori* knowledge of the magnitude of the polarizations, will determine not only the value of most of the helicity amplitudes at small t , but, remarkably, also the values of the beam and target polarizations. This new approach to polarimetry was discussed briefly in [5], but without adequate analysis of the importance of certain correction terms. Subsequently, we have discovered a more direct way of evaluating the polarization from the measured asymmetries, and we have realized that the same method may be applied to colliding beams of different species of spin 1/2 fermions, e.g., p ^3He scattering. We therefore present here a detailed analysis of the pp case using a notation that allows an immediate generalization to the case of non-identical fermions, and we include correction terms which may be important at RHIC energies. We also present a simplified method of obtaining the polarization.

The method we suggest involves the taking of ratios of experimentally determined asymmetries, some of which may be very small. Because so little is known about the helicity amplitudes at high energy we are unable to study this question quantitatively. It will turn out, however, that there are several different ways of determining the polarization of the initial particles and one will have to discover pragmatically which of these provides the most accurate determination. In practice, one would use the values of all the measured asymmetries to extract a best fit to the polarizations.

The method requires the use of asymmetries with both longitudinal and transversely polarized beams. It will not succeed unless data are available with both configurations. Here we work only in order α and so only amplitudes that are large compared to the next order correction can be determined from the formulas given below. (This could probably be improved upon if necessary: at present experiment will probably not be able to probe amplitudes below that size and so we have not pressed on in this direction.) We assume that the polarized beams have the same degree of polarization in either configuration: since they are produced from the same initial configuration by rotation this is almost certainly true.

Although it is likely that the two beams in a pp collider have the same polarization, our formulas will allow for different polarizations P and P' . This distinction will surely be important for beams of different species.

By now there are other methods that will be available for colliding proton beam polarimetry and which may be simpler and more practical than the one we discuss here. Nevertheless, we present the method because it is elegant and it is interesting that P can be determined in this way. Furthermore, it may turn out that this is the best way for ^3He polarimetry, which will very likely be needed eventually at RHIC. Section V presents an alternative method for ^3He polarimetry; it has some model dependence but will almost certainly be practical for high energy p ^3He collisions.

II. IDENTICAL FERMIONS

To see the principles of the method, first consider the elastic collision of identical protons with mass m and anomalous magnetic moment κ . The usual five helicity amplitudes $\Phi_j(s, t)$ [4] are normalized so that

$$\sigma_{\text{tot}}(s) = \frac{4\pi}{\{s(s-4m^2)\}^{1/2}} \text{Im}(\Phi_1 + \Phi_3)_{t=0} \quad (3)$$

and

$$\frac{d\sigma(s, t)}{dt} = \frac{2\pi}{s(s-4m^2)} \{ |\Phi_1|^2 + |\Phi_2|^2 + |\Phi_3|^2 + |\Phi_4|^2 + 4|\Phi_5|^2 \}. \quad (4)$$

At the very small values of t we are interested in, the interference between the strong and electromagnetic forces is crucial, and can be taken into account by writing

$$\Phi_j = \phi_j e^{-i\delta} + \phi_j^{em} \quad (5)$$

where ϕ_i are the hadronic amplitudes and δ is the Coulomb phase that was shown in [7] to be the same for all helicity amplitudes. Its role will be discussed in Sec. IV. In Eq. (5) the ϕ_i^{em} are one-photon exchange amplitudes and are real. We are, of course, primarily interested in learning about the hadronic amplitudes ϕ_i for which we shall write expressions valid for very small t . We define

$$\phi_{\pm} \equiv \frac{1}{2}(\phi_1 \pm \phi_3) \quad (6)$$

so that at high energies $\text{Im } \phi_+(s, 0)$ is large and positive and should be the dominant amplitude. We shall put

$$\phi_+(s, t) = \text{Im } \phi_+(s, 0) (\rho e^{\bar{B}t/2} + i e^{Bt/2}) \quad (7)$$

where $\text{Im } \phi_+$ is short for $\text{Im } \phi_+(s, 0)$, ρ is the usual ratio of real to imaginary parts of the forward spin averaged amplitude and B is the slope of $d\sigma/dt$. Here we allow the possibility that the real and imaginary parts have somewhat different slopes. We also introduce scaled amplitudes as follows:

$$(R_2 e^{\bar{B}_2 t/2} + i I_2 e^{B_2 t/2}) = \frac{\phi_2(s, t)}{2 \text{Im } \phi_+} \quad (8)$$

$$(R_- e^{\bar{B}_- t/2} + i I_- e^{B_- t/2}) = \frac{\phi_-(s, t)}{\text{Im } \phi_+} \quad (9)$$

$$(R_5 e^{\bar{B}_5 t/2} + i I_5 e^{B_5 t/2}) = \left(\frac{m}{\sqrt{-t}} \right) \frac{\phi_5(s, t)}{\text{Im } \phi_+} \quad (10)$$

$$(R_4 e^{\bar{B}_4 t/2} + i I_4 e^{B_4 t/2}) = \left(\frac{m^2}{-t} \right) \frac{\phi_4(s, t)}{\text{Im } \phi_+}. \quad (11)$$

Note the factor of 2 in Eq. (8), which is introduced for later convenience. Note too that we have taken out explicit kinematic factors of $\sqrt{-t}$ and t from ϕ_5 and ϕ_4 , respectively, since for the strong interaction amplitudes $\phi_5 \propto \sqrt{-t}$ and $\phi_4 \propto t$ as $t \rightarrow 0$. The R_j and I_j may vary with s .

We shall need to consider various experimentally determined asymmetries, $PA_N(d\sigma/dt)$, $P'A_N(d\sigma/dt)$, $PP'A_{NN}(d\sigma/dt)$, $PP'A_{LL}(d\sigma/dt)$, $PP'A_{SS}(d\sigma/dt)$ and $PP'A_{SL}(d\sigma/dt)$ [8]. Those contain singular terms as $t \rightarrow 0$ arising from the interference between one-photon exchange and hadronic amplitudes. To order α the asymmetries $A_{NN}(d\sigma/dt)$, $A_{LL}(d\sigma/dt)$ and $A_{SS}(d\sigma/dt)$ are singular as $1/t$ while $A_N(d\sigma/dt)$ and $A_{SL}(d\sigma/dt)$ vary as $1/\sqrt{-t}$. We shall also require the total cross-section differences $PP'\Delta\sigma_T$ and $PP'\Delta\sigma_L$. Keeping therefore the most singular terms as $t \rightarrow 0$ and working to order α , which implies ignoring the Coulomb phase δ in Eq. (5) (the reinsertion of δ will be discussed in Sec. III), we may write the following expressions for the various experimental observables:

$$\Delta_T \equiv -\frac{1}{2} PP' \frac{\Delta\sigma_T}{\sigma_{\text{tot}}} = PP' I_2 \quad (12)$$

TABLE I. Expressions for the coefficients a_j and $b_j^{(0)}$ relevant to the measured observables for pp elastic scattering.

Observable	a_j	$b_j^{(0)}$
\mathcal{I}_0	ρ	$\frac{1}{2}(1 + \rho^2 + R_-^2 + I_-^2) + R_2^2 + I_2^2$
$PP'A_{NN}\mathcal{I}_0$	$PP'R_2$	$PP'[R_2(\rho + R_-) + I_2(1 + I_-)]$
$PP'A_{LL}\mathcal{I}_0$	$PP'R_-$	$PP'[\rho R_- + I_- + R_2^2 + I_2^2]$
$PP'A_{SL}\mathcal{I}_1$	$-PP'\frac{\kappa}{2}(R_2 + R_-)$	$-PP'[R_5(R_2 + R_-) + I_5(I_2 + I_-)]$
$PA_N\mathcal{I}_1$	$P\left[\frac{\kappa}{2}(1 + I_2) - I_5\right]$	$P[-I_5(\rho + R_2) + R_5(1 + I_2)]$
$P'A_N\mathcal{I}_1$	$P'\left[\frac{\kappa}{2}(1 + I_2) - I_5\right]$	$P'[-I_5(\rho + R_2) + R_5(1 + I_2)]$

$$\Delta_L \equiv \frac{1}{2} PP' \frac{\Delta\sigma_L}{\sigma_{\text{tot}}} = PP'I_- . \quad (13)$$

$$\mathcal{I}_1 \equiv \frac{m\sqrt{-t}}{e^{Bt}\sigma_{\text{tot}}} \frac{d\sigma}{dt} . \quad (18)$$

For the differential cross section with slope parameter B for proton-proton collisions, which has an expansion in powers of t beginning with the photon pole and interference terms, we write the form [9]

$$\mathcal{I}_0 \equiv \left(\frac{t}{e^{Bt}\sigma_{\text{tot}}} \right) \frac{d\sigma}{dt} = \frac{4\pi}{\sigma_{\text{tot}}} \frac{\alpha^2}{t} + \alpha a_0 + \frac{\sigma_{\text{tot}}}{8\pi} b_0 t . \quad (14)$$

Here, and in the expressions to follow, it is assumed that coefficients like a_j and b_j (see Table I) are essentially t independent in the very small range of t under consideration. (Note that we have changed the signs of some of the definitions of these quantities from [5] for clarity. Also, there was a sign error in the table entry for A_N in [5] which has been corrected here.) This key assumption can be tested experimentally by confirming that the observables really do follow the t dependence given. The quantities $b_j^{(0)}$ are calculated assuming that all the hadronic slopes are equal and equal to the electromagnetic slopes as well. This will be corrected for below. For asymmetries with initial protons polarized parallel or anti-parallel along the same axis, N (normal to the scattering plane) or L (longitudinal), we have

$$PP'A_{NN}\mathcal{I}_0 = \alpha a_{NN} + \frac{\sigma_{\text{tot}}}{8\pi} b_{NN} t + \dots \quad (15)$$

$$PP'A_{LL}\mathcal{I}_0 = \alpha a_{LL} + \frac{\sigma_{\text{tot}}}{8\pi} b_{LL} t + \dots \quad (16)$$

Spin observables with both initial protons polarized either way along perpendicular axes have a similar power series expansion

$$PP'A_{SL}\mathcal{I}_1 = \alpha a_{SL} + \frac{\sigma_{\text{tot}}}{8\pi} b_{SL} t \quad (17)$$

where again the coefficients a_{SL} , b_{SL} are given in Table I and we have defined

Of course for pp scattering $A_{LS} = A_{SL}$. Note that to the accuracy of our approximations $A_{NN} = A_{SS}$, so the latter is not discussed separately. However, a measurement of A_{SS} could be used as a check on the adequacy of our approximations. For spin asymmetries with only one of the initial protons polarized we have

$$PA_N\mathcal{I}_1 = \alpha a_N + \frac{\sigma_{\text{tot}}}{8\pi} b_N t + \dots , \quad (19)$$

$$P'A_N\mathcal{I}_1 = \alpha a'_N + \frac{\sigma_{\text{tot}}}{8\pi} b'_N t + \dots \quad (20)$$

We write the last pair in this rather pedantic way to emphasize the fact that single spin asymmetries may be measured for either beam and, although the analyzing powers are the same, the polarizations P and P' may be different. For the comparable situation discussed in Sec. IV, the analyzing powers will be different too, in general. The expressions for the measurable coefficients, a_j , b_j , taken from [5], are given in Tables I and II in terms of the hadronic amplitudes R_j , I_j and the anomalous magnetic moment κ . Each b_j is written in the form

$$b_j = b_j^{(0)} + \frac{8\pi\alpha}{\sigma_{\text{tot}}} \Delta b_j \quad (21)$$

with the $b_j^{(0)}$ given in Table I and the Δb_j in Table II. Note that $b_{SS}^{(0)} = b_{NN}^{(0)}$, so it is not listed. We have kept only the largest correction term for b_N . In these expressions we have not included the Coulomb phase δ . We shall explain in Sec. III the consequences of including it. Note that ϕ_4 only occurs in the correction terms Δb_j . In Table II the parameters β_1 , β_2 have the following significance. In order to take into account the t dependence of the electromagnetic form factors of the proton we parametrize the Dirac and Pauli form factors, for very small t , in the form

TABLE II. Correction terms Δb_j in $b_j = b_j^0 + (8\pi\alpha/\sigma_{\text{tot}})\Delta b_j$.

Observable	Δb_j
$PP'A_{NN}\mathcal{I}_0$	$PP'\left[\frac{\kappa^2}{4m^2}\rho + R_2(2\beta_1 + \bar{B}_2/2 - B) - \frac{\kappa}{m}R_5 + \frac{R_4}{2m^2}\right]$
$PP'A_{SS}\mathcal{I}_0$	$PP'\left[\frac{\kappa^2}{4m^2}R_- + R_2(2\beta_1 + \bar{B}_2/2 - B) - \frac{R_4}{2m^2}\right]$
$PP'A_{LL}\mathcal{I}_0$	$PP'\left[R_-(2\beta_1 + \bar{B}_2/2 - B) + \frac{\kappa^2}{4m^2}R_2\right]$
$PP'A_{SL}\mathcal{I}_1$	$-PP'\frac{\kappa}{2}\left[R_2(\beta_1 + \beta_2 + \bar{B}_2/2 - B) + R_-(\beta_1 + \beta_2 + \bar{B}_2/2 - B) - \frac{R_4}{2m^2}\right]$
$PA_N\mathcal{I}_1$	$P\frac{\kappa}{2}(\beta_1 + \beta_2 - B/2)$

$$F_1(t) = e^{\beta_1 t}, \quad F_2(t) = e^{\beta_2 t} \quad (22)$$

where it turns out that with $\Lambda^2 = 0.71 \text{ GeV}^{-2}$ [10],

$$\beta_1 = 2/\Lambda^2 - \kappa/(2m)^2 = 2.31 \text{ GeV}^{-2},$$

$$\beta_2 = 2/\Lambda^2 + 1/(2m)^2 = 3.10 \text{ GeV}^{-2}. \quad (23)$$

Strictly, in order to use our method, one must have information about the slopes \bar{B}_2 and \bar{B}_- , which seem very hard to come by, or else one must neglect these corrections. Since they always appear in the form of a difference from B times one of the small amplitudes it seems reasonable to neglect them in what follows. (Of course the correction from the difference between the electromagnetic slopes and $B/2$ is known and that correction can be applied.)

III. THE METHOD APPLIED TO IDENTICAL PARTICLES

We assume that σ_{tot} , $d\sigma/dt$, ρ , Δ_L and Δ_T [see Eq. (12) and Eq. (13)] are measured, and that some of the coefficients in Eqs. (15) to (20), in particular a_{LL} , b_{LL} and a_{NN} , have been determined. These provide values for the product of the unknown polarizations with the real parts of the non-zero forward amplitudes:

$$\rho = a_o, \quad PP'R_2 = a_{NN}, \quad PP'R_- = a_{LL}. \quad (24)$$

Substituting these results into the expression for b_{LL} in Tables I and II one has

$$\begin{aligned} b_{LL} &= PP' \left[\rho R_- + I_- + R_2^2 + I_2^2 + \frac{2\pi\alpha\kappa^2}{m^2\sigma_{\text{tot}}} R_2 \right] \\ &= \rho a_{LL} + \Delta_L + (a_{NN}^2 + \Delta_T^2)/PP' + \frac{2\pi\alpha\kappa^2}{m^2\sigma_{\text{tot}}} a_{NN} \end{aligned} \quad (25)$$

from which one obtains an expression for the polarization

$$PP' = (a_{NN}^2 + \Delta_T^2) / \left(b_{LL} - \rho a_{LL} - \Delta_L - \frac{2\pi\alpha\kappa^2}{m^2\sigma_{\text{tot}}} a_{NN} \right). \quad (26)$$

There is an important question which we will try to address. How reliable is Eq. (26) if the helicity amplitudes are small? Although we cannot predict the magnitude of the helicity amplitudes at high energies, consideration of quantum number exchange [5] suggests that $|\phi_-|$ will be the smallest of the above hadronic amplitudes, and we expect $|R_-|$, $|I_-| \ll 1\%$ [11]. For the method to work we need $|\phi_2|$ to be considerably larger, albeit still small. How small? In order to use Eq. (26) at all it is clear that one must have the uncertainties $\Delta(b_{LL}) \ll b_{LL}$ and max of $[\Delta(\Delta_L)$ and $\Delta(\Delta_T)] \ll$ either a_{NN} or Δ_T . If this is false then our whole approach would break down even if R_5 and I_5 are large. This is because the reaction then becomes analogous to spin-1/2–spin-0 scattering, for which we certainly cannot determine the amplitudes without knowing P . The requirement that a relative error in PP' less than 10% is obtained in this way is more demanding; for then using Eq. (26) we will need (assuming for illustration that all the needed observables a_i, b_i, Δ_i have the same relative error ϵ and adding the errors in quadrature) $\epsilon < 0.1/\sqrt{5}$. Thus if b_{LL} turns out to be less than 0.1, the error will need to be less than about 0.005 which may be doable, but much smaller will probably not be accessible.

If the first condition is satisfied, but the second is not, so that PP' determined via Eq. (26) has a large uncertainty, there is a more reliable though indirect route to determine the polarization, which involves first determining I_2 and I_5 and then using them to calculate the analyzing power a_N (see Table I). Since they will provide only a small correction to the dominant term $\kappa/2$, it could well be that the errors in their determination are unimportant in A_N .

To achieve this, we assume that we have a rough determination of PP' from Eq. (26) and we use it to provide an improved result. Let us define the combination of observables in Eq. (26) as

$$O_1 \equiv \left(b_{LL} - \rho a_{LL} - \Delta_L - \frac{2\pi\alpha\kappa^2 a_{NN}}{m^2 \sigma_{\text{tot}}} \right) / (a_{NN}^2 + \Delta_T^2). \quad (27)$$

Then from Eq. (24) we have an estimate of the real parts:

$$R_2 = O_1 a_{NN}, \quad R_- = O_1 a_{LL}. \quad (28)$$

Also from Eqs. (12) and (13) we have an estimate of the imaginary parts:

$$I_2 = O_1 \Delta_T, \quad I_- = O_1 \Delta_L. \quad (29)$$

We now proceed to obtain an estimate of R_5 and I_5 . Using a_{SL} and the uncorrected expression for b_{SL} from Table I yields

$$R_5 + \left(\frac{I_2 + I_-}{R_2 + R_-} \right) I_5 = \frac{\kappa}{2} \frac{b_{SL}}{a_{SL}} \quad (30)$$

which, via Eqs. (12), (13) and Table I, becomes

$$R_5 + \left(\frac{\Delta_T + \Delta_L}{a_{NN} + a_{LL}} \right) I_5 = \frac{\kappa}{2} \frac{b_{SL}}{a_{SL}}. \quad (31)$$

The asymmetry A_N provides a second relation between R_5 and I_5 . From Tables I and II we have

$$\frac{b_N}{a_N} = \frac{I_5(\rho + R_2) - R_5(1 + I_2)}{I_5 - \frac{\kappa}{2}(1 + I_2)} + \frac{8\pi\alpha(\beta_1 + \beta_2 - B/2)}{\sigma_{\text{tot}}} \quad (32)$$

where a small term has been neglected in the denominator of the correction term. This can be rewritten as

$$R_5 + \left(\frac{c_N - \rho - R_2}{1 + I_2} \right) I_5 = \frac{\kappa}{2} c_N \quad (33)$$

where

$$c_N \equiv \frac{b_N}{a_N} - \frac{8\pi\alpha(\beta_1 + \beta_2 - B/2)}{\sigma_{\text{tot}}} \quad (34)$$

and the known last term in Eq. (34) may turn out to be negligible. Solving Eqs. (31) and (35) for I_5 and R_5 , using Eq. (29) we obtain

$$I_5 = \frac{\frac{\kappa}{2} \left(\frac{b_{SL}}{a_{SL}} - c_N \right)}{\left(\frac{\Delta_T + \Delta_L}{a_{NN} + a_{LL}} \right) - \left(\frac{c_N - \rho - a_{NN} O_1}{1 + O_1 \Delta_T} \right)} \quad (35)$$

and

$$R_5 = \frac{\kappa}{2} \left\{ \frac{b_{SL}}{a_{SL}} - \frac{\frac{b_{SL}}{a_{SL}} - c_N}{1 - \frac{a_{NN} + a_{LL}}{\Delta_T + \Delta_L} \frac{c_N - \rho - a_{NN} O_1}{1 + \Delta_T O_1}} \right\}. \quad (36)$$

Now if it turns out that I_5 is very small, say 1% or smaller, then the neglect of the correction term in the expression for b_{SL} renders our estimate inaccurate. However, I_5 is then so small compared to $\kappa/2$ in the expression for a_N that it is irrelevant and should be neglected. If, on the other hand, it is “large,” i.e., $\approx 10\%$, then the neglected correction term in b_{SL} is negligible and the estimate for I_5 (and R_5) can be used in computing the hadronic corrections to A_N . As stressed earlier, since I_5 and I_2 are small corrections compared to $-\kappa/2$ in a_N , the errors on them, even if relatively large—and they may be, given the large number of measured quantities that go into Eq. (35) and Eq. (36)—should not be important in computing the value of a_N . Once A_N is known the polarization can be measured in the normal way.

Up to the present we have ignored the Coulomb phase δ which appears in Eq. (5). We now consider the effect of including it. It will turn out that δ has *no* effect upon the determination of the polarization given in Sec. III. Its only role will be in the evaluation of the hadronic amplitudes themselves. It is given by [15–17]

$$\delta = -\alpha \{ \ln[|t|(B/2 + 4/\Lambda^2)] + \gamma \} \quad (37)$$

where B is the logarithmic slope of the pp $d\sigma/dt$ at $t=0$, $\Lambda^2 = 0.71 \text{ GeV}^2$ and γ is Euler's constant $\gamma = 0.5772$. Experimentally $B \approx 13 \text{ GeV}^{-2}$ at high energies and increases slowly through the RHIC energy range. The phase δ is very small except at extremely tiny values of $|t|$, so that we can take

$$\phi_j e^{-i\delta} \approx (\text{Re } \phi_j + \delta \text{Im } \phi_j) + i \text{Im } \phi_j. \quad (38)$$

Thus to take into account δ one must make the replacements

$$R_j \rightarrow R_j + I_j \delta \quad (39)$$

and

$$\rho \rightarrow \rho + \delta \quad (40)$$

throughout Secs. II and III. Note that, strictly speaking, δ is a function of t . However, it seems hopeless to try to detect the $\ln|t|$ behavior in Eq. (37), so we suggest that Eq. (39) and Eq. (40) should be used with an effective t -independent $\bar{\delta}$, equal to the mean value of $\delta(t)$ for the region of t covered experimentally. It is important to note that the substitutions Eq. (39) and Eq. (40) do *not* affect the result Eq. (26) for PP' , nor the result for A_N calculated on the basis of the Table I and II formula, in which the results Eq. (35), Eq. (36), Eq. (28) and Eq. (24) are used. The phase δ only plays a role in the determination of the hadronic amplitudes *per se*, i.e., any amplitude derived via Eq. (24), Eq. (28), Eq. (35) and Eq. (36) must be reinterpreted on the basis of Eq. (39) and Eq. (40).

IV. NONIDENTICAL SPIN HALF FERMIONS

The above analysis generalizes to the case of the collision of non-identical spin half particles, protons colliding with ^3He in the CNI region, for example. It may be that the self-

calibration of proton polarization through the use of proton-proton CNI collisions is difficult at particular energies because of unsuitably small spin asymmetries. In such cases it is, in principle, possible to evaluate the proton polarization by a study of the non-identical collision process.

Suppose that the second distinct fermion of mass m' and polarization P' has charge Ze , and anomalous magnetic moment κ' , given in units of proton magnetons, as is usual for nuclei. Real and imaginary scaled amplitudes are introduced as before that now involve the center of mass momentum k defined by $4k^2s = [s - (m' - m)^2][s - (m' + m)^2]$. A distinct helicity flip amplitude for the second fermion

$$(R_6 e^{\bar{B}_6 t/2} + i I_6 e^{B_6 t/2}) = \left(\frac{m}{\sqrt{-t}} \right) \frac{\phi_6(s, t)}{\text{Im } \phi_+} \quad (41)$$

appears in non-identical elastic collisions.

Using only initial state polarization, with one or both beams polarized, one can measure seven spin-dependent asymmetries. We follow the notation of [7]; notice that ϕ_5 describes the proton spin flip while ϕ_6 describes the ^3He spin flip, while A_N is the proton analyzing power and A'_N is the ^3He analyzing power.

$$\begin{aligned} \frac{d\sigma}{dt} &= \frac{\pi}{2k^2s} \{ |\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 2|\phi_5|^2 \\ &\quad + 2|\phi_6|^2 \} \\ A_N \frac{d\sigma}{dt} &= -\frac{\pi}{k^2s} \text{Im}[(\phi_1 + \phi_3)\phi_5^* - (\phi_2 - \phi_4)\phi_6^*] \\ A_{NN} \frac{d\sigma}{dt} &= \frac{\pi}{k^2s} \text{Re}[\phi_1\phi_2^* - \phi_3\phi_4^* - 2\phi_5\phi_6^*] \\ A'_N \frac{d\sigma}{dt} &= \frac{\pi}{k^2s} \text{Im}[(\phi_1 + \phi_3)\phi_6^* - (\phi_2 - \phi_4)\phi_5^*] \\ A_{SS} \frac{d\sigma}{dt} &= \frac{\pi}{k^2s} \text{Re}[\phi_1\phi_2^* + \phi_3\phi_4^*] \\ A_{SL} \frac{d\sigma}{dt} &= \frac{\pi}{k^2s} \text{Re}[(\phi_2 + \phi_4)\phi_5^* - (\phi_1 - \phi_3)\phi_6^*] \\ A_{LS} \frac{d\sigma}{dt} &= \frac{\pi}{k^2s} \text{Re}[(\phi_1 - \phi_3)\phi_5^* - (\phi_2 + \phi_4)\phi_6^*] \\ A_{LL} \frac{d\sigma}{dt} &= \frac{\pi}{2k^2s} \{ |\phi_1|^2 - |\phi_3|^2 + |\phi_2|^2 - |\phi_4|^2 \}. \end{aligned} \quad (42)$$

For the scattering of a proton on a spin-1/2 fermion of charge Z and anomalous magnetic moment κ' the electromagnetic amplitudes are approximately, for large s and small t ,

$$\begin{aligned} \phi_1^{\text{em}} &= \phi_3^{\text{em}} = \frac{\alpha s Z}{t} F_1 F'_1 \\ \phi_2^{\text{em}} &= -\phi_4^{\text{em}} = \frac{\alpha s \kappa \kappa'}{4m^2} F_2 F'_2 \\ \phi_5^{\text{em}} &= -\frac{\alpha s \kappa Z}{2m\sqrt{-t}} F'_1 F_2 \\ \phi_6^{\text{em}} &= \frac{\alpha s \kappa'}{2m\sqrt{-t}} F_1 F'_2 \end{aligned} \quad (43)$$

where $\mu_p = \kappa + 1$ is the proton's magnetic moment, and m its mass. We recall that the proton electromagnetic form factors are denoted $F_1(t)$ and $F_2(t)$ while the Dirac and Pauli form factors of ^3He we will describe by $F'_1(t)$ and $F'_2(t)$. All of the form factors are normalized to 1 at $t=0$. The form factor F'_1 appears multiplied by the charge of ^3He , $Z=2$, but F'_2 does not. Rather it appears with the factor κ' ; note that the magnetic moments are given in units of nuclear magnetons, as is normal for nuclei, and so the proton mass m appears in the denominator of the expression for ϕ_6^{em} in Eq. (43) and in the numerator of Eq. (41).

\mathcal{I}_0 and \mathcal{I}_1 are defined as before, and

$$\mathcal{I}_0 \left(\frac{t}{e^{Bt} \sigma_{\text{tot}}} \right) \frac{d\sigma}{dt} = \frac{4\pi}{\sigma_{\text{tot}}} \frac{Z^2 \alpha^2}{t} + \alpha a_0 + \frac{\sigma_{\text{tot}}}{8\pi} b_0 t. \quad (44)$$

In addition to A'_N , which is truly independent of A_N here, one new non-trivial spin observable for the case of distinct fermions involves A_{LS} in addition to A_{SL} , with an additional equation corresponding to Eq. (17)

$$PP' A_{LS} \mathcal{I}_1 = \alpha a_{LS} + \frac{\sigma_{\text{tot}}}{8\pi} b_{LS} t. \quad (45)$$

The analysis proceeds as in the identical particle case up to the point just before consideration of the evaluation of R_5 . To determine values of R_5 and R_6 , now we observe from the expressions for A_{SL} and A_{LS} in Table III that

$$\begin{aligned} a_{SL} \pm a_{LS} &= -PP' \frac{1}{2} (\kappa Z \pm \kappa') (R_2 \pm R_-) \\ b_{SL} \pm b_{LS} &= -PP' (R_5 \mp R_6) (R_2 \pm R_-) \\ &\quad - PP' (I_5 \mp I_6) (I_2 \pm I_-). \end{aligned} \quad (46)$$

Upon division, we are led to two linear relations among the four quantities R_5 , R_6 , I_5 , and I_6 so that from Eq. (46)

$$(R_5 \mp R_6) + (I_5 \mp I_6) \frac{I_2 \pm I_-}{R_2 \pm R_-} = \frac{1}{2} (\kappa Z \pm \kappa') \frac{b_{SL} \pm b_{LS}}{a_{SL} \pm a_{LS}}. \quad (47)$$

Two further linear relations among R_5 , R_6 , I_5 , and I_6 may be obtained from consideration of A_N and A'_N :

TABLE III. The left column shows the reaction parameter, an asymmetry in all cases excepting the first, the unpolarized cross section. The second and third columns indicate the related expressions for the coefficients a_T and b_T .

Observable	a_T	b_T
\mathcal{I}_0	$Z\rho$	$\frac{1}{2}(1+\rho^2+R_-^2+I_-^2)+R_2^2+I_2^2$
$PP'A_{NN}\mathcal{I}_0$	$PP'ZR_2$	$PP'[R_2(\rho+R_-)+I_2(1+I_-)]$
$PP'A_{LL}\mathcal{I}_0$	$PP'ZR_-$	$PP'(\rho R_-+I_-+R_2^2+I_2^2)$
$PP'A_{SL}\mathcal{I}_1$	$-PP'(\frac{1}{2}\kappa ZR_2+\frac{1}{2}\kappa'R_-)$	$PP'(-R_2R_5-I_2I_5+R_-R_6+I_-I_6)$
$PA_N\mathcal{I}_1$	$P(\frac{1}{2}\kappa Z+\frac{1}{2}\kappa'I_2-ZI_5)$	$P(-\rho I_5+R_5+R_2I_6-I_2R_6)$
$PP'A_{LS}\mathcal{I}_1$	$-PP'(\frac{1}{2}\kappa'R_2+\frac{1}{2}\kappa ZR_-)$	$PP'(R_2R_6+I_2I_6-R_-R_5-I_-I_5)$
$P'A'_N\mathcal{I}_1$	$P'(\frac{1}{2}\kappa'+\frac{1}{2}\kappa ZI_2+ZI_6)$	$P'(\rho I_6-R_6-R_2I_5+I_2R_5)$

$$R_5-I_2R_6-\rho I_5+R_2I_6=\frac{1}{2}(\kappa Z+\kappa'I_2-2ZI_5)\frac{b_N}{a_N}, \quad (48)$$

$$I_2R_5-R_6-R_2I_5+\rho I_6=\frac{1}{2}(\kappa'+\kappa ZI_2+2ZI_6)\frac{b'_N}{a'_N}. \quad (49)$$

From the four linear relations Eq. (47), (48), (49) values of R_5 , R_6 , I_5 , and I_6 may be found in general, just as in Eq. (35) and Eq. (36). A glance at PA_N and $P'A'_N$ in Table III reveals that the equations

$$a_N=P\left(\frac{1}{2}\kappa Z+\frac{1}{2}\kappa'I_2-ZI_5\right) \quad (50)$$

$$a'_N=P'\left(\frac{1}{2}\kappa'+\frac{1}{2}\kappa ZI_2-ZI_6\right) \quad (51)$$

provide estimates of the individual polarizations, P and P' , of the two distinct fermions participating in the elastic collision. All the ingredients for successful polarimetry are to hand. If the various coefficients a_I and b_I involved in the evaluation of the scaled amplitudes are sufficiently non-zero within error, then Eq. (50) indicates the extent of the asymmetry maximum to be expected for polarized protons. The corrections and error discussion go through just as in Sec. III.

V. AN ALTERNATIVE METHOD FOR ^3He POLARIMETRY

Although the method described here is elegant and totally model independent, it may fail if the key asymmetries turn out to be too small to measure precisely. In this section, we present an alternative method which depends only on measuring A_N and A'_N , asymmetries which will almost certainly be large enough to measure accurately. The price to pay will be that some model dependence will enter, but this dependence can probably be controlled and corrected for [12–14].

To a first approximation, the amplitude for flipping the spin of ^3He is the same as that for flipping the neutron spin in pn elastic scattering. So we begin with the simple parametrization of the pn amplitudes that corresponds to that used

previously for the pp amplitudes [5]:

$$\phi_+(s,t)=\frac{s}{8\pi}\sigma_{\text{tot}}^{pn}(i+\rho^{pn})e^{B^{pn}t/2},$$

$$\phi_5(s,t)=\frac{\tau_p\sqrt{-t}}{2m}\phi_+(s,t),$$

$$\phi_6(s,t)=-\frac{\tau_n\sqrt{-t}}{2m}\phi_+(s,t). \quad (52)$$

We will neglect ϕ_- , ϕ_2 and ϕ_4 . See [5] for a discussion of this approximation. In general, τ_p and τ_n are complex, but Regge arguments favor nearly real values for high energy. (There is a factor of 2 different in the convention for τ from [5]. This is chosen to parallel more closely the anomalous magnetic moment in the electromagnetic amplitudes.)

We will now make several approximations in order to keep the discussion simple. They can be corrected for if a more precise calculation is needed to interpret experiment. First, we assume that we are at sufficiently high energy that the scattering is given by pure $I=0$ exchange. This is almost certainly true at RHIC but corrections can be made when using the approach at lower energy [18]. Thereby we will have $\tau_p=\tau_n\equiv\tau$ and the other parameters $\sigma_{\text{tot}}^{pn}, \rho^{pn}, B^{pn}$ will have the values of the corresponding pp parameters. Next, we will assume independent particle harmonic oscillator wave functions for the nucleons in ^3He with the two protons coupled to spin singlet so all the spin is carried by the neutron. Finally, we will use the impulse approximation for calculating the $p\ ^3\text{He}$ scattering in terms of pn and pp scattering. These aspects can easily be improved upon by using better wave functions and multiple scattering theory [14]. The results here should not be misleading at the very small t values of the CNI peak but the corrections should be calculated if data at larger values of $|t|\geq 0.1\text{ GeV}^2$ are used in fitting data to the calculated curve.

Thus we obtain for the hadronic part of the $p\ ^3\text{He}$ amplitudes

$$\begin{aligned}
\phi_+^{p^3\text{He}}(3s, t) &= \frac{9s}{8\pi} \sigma_{\text{tot}}(s)(i + \rho) F_H(t), \\
\phi_5^{p^3\text{He}}(3s, t) &= \frac{\tau\sqrt{-t}}{2m} \phi_+^{p^3\text{He}}(3s, t), \\
\phi_6^{p^3\text{He}}(3s, t) &= -\frac{1}{3} \frac{\tau\sqrt{-t}}{2m} \phi_+^{p^3\text{He}}(3s, t). \quad (53)
\end{aligned}$$

Note here that s denotes the s value for the pn or pp subsystem and σ_{tot} denotes the pp total cross section, so $\sigma_{\text{tot}}^{p^3\text{He}}(3s) = (8\pi/3s) \text{Im} \phi_+(3s, 0)$. The factor 3 in the first equation (and the corresponding 1/3 in the last) results from there being three nucleons in ^3He (but only one neutron). $F_H(t) = \exp\{t(B/2 + a^2/4)\}$ where a^2 is the harmonic oscillator parameter, approximately equal to 57.4 GeV^{-2} .

In order to calculate the CNI effect, we need also the electromagnetic amplitudes for $p^3\text{He}$ scattering. These are (we suppress an index referring to the $p^3\text{He}$ for typographical clarity)

$$\begin{aligned}
\phi_+^{em}(3s, t) &= \frac{6s\alpha}{t} F_{em}(t), \\
\phi_5^{em}(3s, t) &= -\frac{6s\alpha}{2m\sqrt{-t}} \kappa_p F_{em}(t), \\
\phi_6^{em}(3s, t) &= \frac{3s\alpha}{2m\sqrt{-t}} \kappa_n F_{em}(t), \quad (54)
\end{aligned}$$

where $F_{em}(t) = \exp(a^2 t/4)$. Note that m in all these formulas denotes the proton mass. In the third of these equations one might want to use the magnetic moment of ^3He rather than κ_n ; this is about 10% larger in magnitude. We leave the expressions this way for consistency with our simple approach.

Using these relations one can calculate the quantities a_N , a'_N , b_N , b'_N as well as a_0 and b_0 . Normalizing to the total cross section $A\sigma_{\text{tot}}$ where A is the nuclear number we get

$$a_0 = Z\rho, \quad (55)$$

$$b_0 = \frac{1}{2} A(1 + \rho^2), \quad (56)$$

$$a_N = \frac{1}{2} P Z [\kappa_p - \text{Re}(\tau) - \rho \text{Im}(\tau)], \quad (57)$$

$$b_N = \frac{1}{2} P A \text{Im}(\tau)(1 + \rho^2), \quad (58)$$

$$a'_N = \frac{1}{2} P' \left(\kappa_n - \frac{Z}{A} [\text{Re}(\tau) + \rho \text{Im}(\tau)] \right), \quad (59)$$

$$b'_N = \frac{1}{2} P' \text{Im}(\tau)(1 + \rho^2). \quad (60)$$

Here, of course, $A=3$ and $Z=2$; we write the expressions in this way so that the source of the factors which differ from the pp case is transparent.

The important result that b_N and b'_N are simply related to each other results from the assumption of $I=0$ dominance of both the flip and non-flip amplitudes. If one knew the proton polarization P from an independent polarimeter, then this simple relation immediately gives P' . It is more in the spirit of the present work to use this one experiment to determine everything. Thus

$$\frac{b_N}{b'_N} = A \frac{P}{P'} \equiv \mathcal{M}_2. \quad (61)$$

This is to be used then with

$$\frac{a_N}{a'_N} = \frac{Z[\kappa_p - \text{Re}(\tau) - \rho \text{Im}(\tau)]}{\kappa_n - \frac{Z}{A} [\text{Re}(\tau) + \rho \text{Im}(\tau)]} \frac{P}{P'} \equiv \mathcal{M}_1 \quad (62)$$

to solve for $\text{Re}(\tau[1 - i\rho])$ in terms of these two measurable ratios \mathcal{M}_1 and \mathcal{M}_2 :

$$\frac{Z}{A} \text{Re}(\tau[1 - i\rho]) = \frac{\kappa_n \mathcal{M}_1 - \frac{Z}{A} \kappa_p \mathcal{M}_2}{\mathcal{M}_1 - \mathcal{M}_2}. \quad (63)$$

With this result in hand, one can use a_N to obtain P and a'_N to obtain P' . The only fly in the ointment—and it is a pretty serious fly—is that if $\text{Im}(\tau)$ should vanish the method fails at step one because both b_N and b'_N then vanish. Regge theory suggests that this is likely to happen at sufficiently high energy, but at RHIC there may remain a sufficient phase difference between the flip and non-flip amplitudes that both b_N and b'_N are measurably finite. If $\text{Im}(\tau)$ does vanish, then this method can be used as a polarimeter for ^3He in conjunction with an independent measurement of the proton beam polarization P : for then one can use Eq. (59) to obtain τ and use it in Eq. (60) to obtain P' .

VI. CONCLUSIONS

This paper presents a model-independent method for determining the polarization of colliding spin-1/2 particles, like or unlike, based on measuring the various single- and double-spin asymmetries using both longitudinal and transversely polarized beams. No independent information regarding the polarization is needed. The method is dependent on precise measurements of several probably very small quantities, and this may limit its applicability, but the existence of the method is interesting in itself. Finally, an alternative method is given, which depends on some nuclear physics and some high energy approximations, but which should almost certainly be practical for unlike particles.

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